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The Principle of Proportionality of Mass and Energy: New Version

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The essence of mass and its relation to the relativistic energy is considered. It is assumed that the rest energy is equal to the total binding energy of the body and can be found through the energies of fundamental fields associated with the substance of the body. Mass as a measure of inertia is calculated by relativistic energy and relativistic momentum. The conclusion is made that after radiation of energy from a system the mass of the system must not decrease, but increase. The opposite case is heating of bodies by external sources, which must be accompanied by an increase in entropy and decrease in the mass of the bodies. On the basis of strong gravitation the mass defect of atomic nuclei is explained. Conclusions of the general theory of relativity and the covariant theory of gravitation with respect to the mass and energy of gravitational field are opposite – in the general theory of relativity, relativistic energy and mass of a body are reduced by the mass-energy of its own gravitational field, and in the covariant theory of gravitation the mass-energy of the gravitational field increases the relativistic energy and body mass.

Key words: Mass, Energy, Principle of equivalence, Tensor of entropy, Mass defect

1. INTRODUCTION

Knowledge of the essence of mass and energy, as well as ways of defining them, are one of the most important problems in physics. This is due to extensive use of the law of conservation of energy and momentum in various areas and due to the possibility of calculating the acting forces through the energy gradients in spacetime. The relationship between mass and energy should be the most simple in the frame of reference in which the body rests and does not rotate, since the momentum and the angular momentum of the body are equal to zero and the kinetic energy of linear motion of the body as a whole and the rotational energy do not participate in the calculation of the mass.

2. METHODS

Studying the relationship of mass and energy of a body at rest during the formation of the theory of relativity led to the relation:

$$\Sigma_0 = kmc^2, \qquad (1)$$

where Σ_0 – the relativistic energy at rest, m – the body mass, c – the speed of light.

Initially it was considered that the coefficient k in (1) is sufficiently close to 1, then by the efforts of O. Heaviside in1889 (Болотовский Б М 1985), and a number of other physicists (Poincare A 1900; Einstein A 1905) it was established that $k = 1$.

From (1) it does not follow that the mass and the relativistic rest energy are synonyms, the designation of the same. On the one hand, the body mass is an integral property that determines the inertia of the body upon receipt of acceleration from a force. Integrality here means that not only substance of the body makes contribution to the mass but also physical fields associated with this substance, as well as fields from external sources in the body volume. On the other hand, the energy is associated more with the law of conservation of energy, with the ability to transfer of energy from one place to another in different ways and in different forms, such as heat transfer, electromagnetic radiation, electricity, etc. The force $\boldsymbol{F}$, acting on the body, is defined as the rate of change of momentum, and the mass is directly included in the momentum $\boldsymbol{P}$ as a factor. When calculating the force in external field the formula is often used in which the force is the gradient of potential energy U of the body, taken with opposite sign. This implies the following:

$$\boldsymbol{F} = \frac{d\boldsymbol{P}}{dt} = -\nabla U \, . \qquad (2)$$

It is seen from (2) that although all forms of energy are contributing to the body mass (and hence to the momentum), but the force as the rate of change of momentum can only depend on the gradients of certain energies. If we take the average over the entire volume, the energies without gradients and energy fluxes do not cause force and acceleration, although they are involved in formation of the body mass. In this connection it must be assumed that the mass represents the static integral gravitational and inertial properties of the body, appearing as a result of energy fluxes that interact with the body.

3. RESULTS AND DISCUSSION

3.1. Mass and energy in thought experiment

As it was described in (Einstein A 1905) the body mass emitting some energy L in the form of two oppositely directed photons, should be reduced by the amount $\Delta m = L / c^2$. To verify this conclusion, we once again repeat the thought experiment with the body, which emits photons. In this case we shall use the formulas for relativistic energy and momentum from (Fedosin S G 2011a):

$$\Sigma = \frac{\Sigma_0}{\sqrt{1-V^2/c^2}} \, , \qquad \boldsymbol{P} = \frac{\Sigma_0 \boldsymbol{V}}{c^2\sqrt{1-V^2/c^2}} \, , \qquad \Sigma_0 = -E_{g0} - E_0 \, , \qquad (3)$$

where Σ_0 is a positive value for the relativistic energy of a body at rest, E_{g0} is the total energy of particles of the body at the atomic level, which includes various types of energies associated with atoms and molecules near absolute zero temperature: the energy of strong interaction, which bounds the substance of elementary particles and retains the nucleons in atomic nuclei; the energy of electromagnetic interaction of particles; the energy of motion of substance inside the nucleons and the energy of the nucleons in nuclei and the electrons in atoms; the rotational energy of atoms and molecules; the vibrational energy of atoms in molecules, etc.,

E_0 is the total energy of the body at the macro level, taking into account the internal kinetic energy E_k in the form of kinetic energy of chaotic motion of atoms and molecules and the energy of the turbulent motion of substance fluxes, as well as the energies of the fundamental macroscopic fields.

The sum $E_{g0} + E_k$ is the internal energy of a system, commonly used in thermodynamics in the weak-field interaction between the particles of the system. According to (3), the relativistic energy is binding energy of the body, or the total energy taken with the negative sign. By definition, the total macroscopic energy E_0 can be divided into three components:

$$E_0 = E_k + U_0 + W_0 \, , \qquad (4)$$

where U_0 and W_0 are the energies of macroscopic gravitational and electromagnetic fields in the substance of the body, respectively, calculated inside the body and outside of it. In the same energies the energies of fields from external sources should be included that fall inside the body and change the energy of the substance.

We shall take into account in (4) the virial theorem, according to which the absolute value of the potential energy of fields is on the average twice as much than the internal kinetic energy for bodies that are only under action their proper gravitational and electromagnetic fields:

$$2E_k + U_0 + W_0 \approx 0 \, , \qquad E_0 = E_k + U_0 + W_0 \approx -E_k \approx \frac{U_0 + W_0}{2} \, . \qquad (5)$$

For the energy E_{g0} similarly to (5) it can be written down:

$$E_{g0} \approx \frac{U_{g0} + W_{g0}}{2} \, , \qquad (6)$$

where U_{g0} is the total energy of the field of strong gravitation, which is assumed at the level of elementary particles and atoms, and instead of strong interaction holds the substance of nucleons and the nucleons in atomic nuclei, as well as is one of the components that holds electrons in their orbits in atoms,

W_{g0} – is the electromagnetic energy in the substance of elementary particles and around them in atoms.

Taking into account (5) and (6) the relativistic energy (3) of a body at rest can be written down as follows:

$$\Sigma_0 = -E_{g0} - E_0 = -\frac{U_{g0} + W_{g0}}{2} - \frac{U_0 + W_0}{2} . \qquad (7)$$

The main contribution to the energy Σ_0 is made by the negative energy U_{g0} of the field of strong gravitation, which provides the positivity of the relativistic energy and the body mass, determined by the expression:

$$m = \frac{\Sigma_0}{c^2} . \qquad (8)$$

We shall pass now to the thought experiment. We shall suppose there is a body at rest and two photons are emitted from the body in opposite directions, one photon with the energy E_F along the axis OX, and the other with the same energy against the axis OX. We shall assume that the photon emission occurs only due to changes in the energy of macroscopic fields, then the energy balance before and after the emission has the form:

$$\Sigma_0 = -E_{g0} - E_0 = -E_{g0} - E_1 - 2E_F = \Sigma_1 - 2E_F . \qquad (9)$$

After the emission the body remains stationary, since the momenta of the photons are opposite and the total momentum of the system remains zero. It is assumed in (9) that at the moment of emission the components of the energy Σ_0 are changed, and the energy of photons is taken with the minus sign, which means the body's energy loss due to emission. It follows that $E_0 - E_1 = 2E_F$, and hence $E_0 > E_1$. Both energies E_0 and E_1 are negative, so that for the absolute values of energy we obtain: $|E_1| > |E_0|$. Relation (9) also shows that the relativistic energy Σ_1 is greater than the energy Σ_0.This entails an increase in the body mass after the emission of photons: $m_1 = \frac{\Sigma_1}{c^2} > m = \frac{\Sigma_0}{c^2}$. Such result conforms to the fact that the more energy a star radiates, the more this star is compressed and heated. The total energy E_0 of such star becomes more negative and the positive internal kinetic energy E_k in (5) increases, which according to (7) and (8) increases the mass of the star. One of the reasons for choosing the negative sign before E_0 in (7) is the symmetry of this expression when E_{g0} has the negative sign too. In addition, the energy of gravitation in the compression of the star is converted into the internal kinetic energy and the radiation energy, and according to virial theorem these energies are approximately equal to each other. If the energy of gravitation generates the radiation energy, in the same degree the energy of gravitation can generate the additional relativistic energy and mass of the star. In this case, the system star + universe obeys the law of energy conservation – the negative change of the gravitational energy in the compression of the star is compensated by a positive change in the internal energy of the substance and the appearance of the radiation energy from the star.

In principle, electromagnetic radiation and photon emission are impossible, if no work is done on the electric charges. If a charge is accelerated by the gravitational force, the work of gravitation will increase the energy of the system with transfer of some energy to radiation from the charge. In betatron the work of the magnetic field is converted into acceleration of electrons producing synchrotron emission. In atom, the strong gravitation and the electric field of the nucleus do work on the electron in its transition from one energy state to another, which leads to emission from the atom. In all cases, the work performed exceeds the energy of radiation, which makes it possible to increase the relativistic energy of radiating system at the time of radiation.

The described above grounds for (9) and for choosing the negative sign before the photon energy $2E_F$ are missing in (Einstein A 1905). Instead of it the mechanical model of the phenomenon is considered, when the photons as some parts of the body leave the body and take away part of its mass. Accordingly, in this picture the positive sign of the photon energy is chosen, and the body mass after the emission of photons should decrease. However, the photons are not part of the body because they are generated due to absolute acceleration of charges of the body without decrease in the magnitude of these charges (if the body mass can vary due to changes in the body's energy, then the charge of the body remains until the moment when it will removed from the body or compensated by a charge of the opposite sign). Therefore, the loss of the relativistic energy of the body due to the transfer of photon energy must be compensated by an increase rather than decrease in the energy and the body mass.

We shall now consider the photon emission from the body moving at the velocity V along the axis OX. The photon emitted in the direction of the axis OX, will have blue shift of its wavelength and the increased energy, and the photon emitted in the opposite direction, will have red shift of the wavelength and the decreased energy. The total energy of both photons according to formula for the Doppler effect will be equal to $2E'_f = \frac{2E_F}{\sqrt{1-V^2/c^2}}$, and the total momentum of the photons is equal to $P_F = \frac{2E_F V}{c^2\sqrt{1-V^2/c^2}}$ and is directed along the velocity of the body.

Taking into account the formulas (3), the balance of energies and momenta before and after the photon emission gives the following:

$$\Sigma = \frac{\Sigma_0}{\sqrt{1-V^2/c^2}} = \frac{\Sigma_1}{\sqrt{1-V^2/c^2}} - 2E'_f = \frac{\Sigma_1}{\sqrt{1-V^2/c^2}} - \frac{2E_F}{\sqrt{1-V^2/c^2}},$$

$$P = \frac{\Sigma_0 V}{c^2\sqrt{1-V^2/c^2}} = \frac{\Sigma_1 V}{c^2\sqrt{1-V^2/c^2}} - P_F = \frac{\Sigma_1 V}{c^2\sqrt{1-V^2/c^2}} - \frac{2E_F V}{c^2\sqrt{1-V^2/c^2}}. \qquad (10)$$

In (10) the energy and momentum of photons have the minus sign, since the photons carry away from the body some part of its energy and momentum. In the moment of photon emission a corresponding increase in body mass, relativistic energy and momentum takes place. After canceling the identical terms (10) turns into (9). This means that the difference between the formulas for the processes of photon emission of the body at rest and the body in motion is associated only with the Lorentz transformation and is determined by the factor $\sqrt{1-V^2/c^2}$.

3.2. Heating of bodies

From the above we can come to the idea that heating of a body by the external sources of energy should decrease the body mass. As it was found in (Fedosin S G 1999) based on Lorentz-invariant thermodynamics, the amount of heat δQ, that is appeared in a certain volume V_b of the body during the time dt, is determined by the integral:

$$\delta Q = -dt\int \nabla\cdot(\boldsymbol{S}_\Gamma + \boldsymbol{S}_P)\, dV_b = -dt\int (\boldsymbol{S}_\Gamma + \boldsymbol{S}_P)\cdot \boldsymbol{n}\, dS_b, \qquad (11)$$

where $\boldsymbol{S}_\Gamma$ is the density of the flux of gravitational energy, $\boldsymbol{S}_P$ – the electromagnetic energy flux density (the Poynting vector), $\boldsymbol{n}$ – the unit vector of the normal to the surface area S_b surrounding the volume V_b.

According to (11), the increase in the heat can be described by the incoming fluxes of energy of the fundamental fields – either by the integral of the divergences of energy fluxes over volume or by using the Gauss theorem for the integral of the energy fluxes over the area. Equation (11) is easier to understand if we consider the following formulas:

$$\nabla\cdot\boldsymbol{S}_\Gamma = -\frac{\partial U^{00}}{\partial t} - \boldsymbol{J}\cdot\boldsymbol{G}, \qquad \nabla\cdot\boldsymbol{S}_P = -\frac{\partial W^{00}}{\partial t} - \boldsymbol{j}\cdot\boldsymbol{E}, \qquad (12)$$

where U^{00} and W^{00} are the energy densities of gravitational and electromagnetic fields in the form of timelike components of the corresponding stress–energy tensors, $\boldsymbol{J}$ and $\boldsymbol{j}$ – the densities of mass and electric current, respectively, $\boldsymbol{G}$ and $\boldsymbol{E}$ – the strengths of gravitational and electromagnetic fields (gravitational acceleration and electric strength).

If we substitute (12) in (11), we see that the heat in volume of the body increases when the energy of field is increasing, as well as when due to the energy of fields the work $\boldsymbol{J}\cdot\boldsymbol{G}+\boldsymbol{j}\cdot\boldsymbol{E}$ is done in the unit volume per unit time. The differential of entropy is expressed by the formula:

$$dS=\frac{\delta Q}{T}, \qquad (13)$$

where T is the absolute Kelvin temperature.

According to (13), if the body is heated by the external sources, the entropy of the body increases. If the energy is radiated from the body in the process of gravitational contraction and heating of the substance, the total energy of the body is reduced by δQ and the increment of entropy dS is negative. This is due to the fact that although the substance under compression and decrease in its volume is heated and the entropy of the substance increases, but the negative entropy of the gravitational field of the body changes even more, so that the total entropy of the substance and the field is negative. For the entropy of a spherical body, we derived the formula (Fedosin S G 1999):

$$S=-\int\frac{\boldsymbol{r}\cdot\nabla(U^{00}+W^{00}+L-P_0)}{T}\,dV_s\,, \qquad (14)$$

where the radius-vector $\boldsymbol{r}$ is measured from the center of the body, P_0 – the pressure in the comoving reference frame, $L=\int\frac{P_0}{\rho_0}\,d\rho_0$ is the function of compression, calibrated so that the energy density of the substance at rest is equal to the value $\rho_0 c^2$, ρ_0 – the density of substance at rest.

In (14) the integration is over the entire volume V_s of space, both inside and outside the body. The main contribution to the negative entropy of the body is made by the gradient of the gravitational field energy density U^{00} and the gradient of pressure P_0. Estimation of entropy per particle of ideal gas in gravitationally bound ball at a constant temperature of the volume, gives the value $\approx -7.2\,k$, where k is the Boltzmann constant.

As the energy is radiated from the body the entropy of the body becomes more negative, the entropy of the outgoing radiation is positive, in the result the total entropy of the body and the radiation is zero. This conclusion follows from the virial theorem and from (13), in which δQ means both the heat content of the body as a result of its gravitational contraction, and the energy carried away by the outgoing radiation. Zero entropy was at the beginning of formation of the body too when the substance at infinity was at rest and in the dispersed state.

In (Fedosin S G 2009a), we have derived the Lorentz covariant expression of the first law of thermodynamics, have found the tensor function of the chemical potential, the tensor function of the work-energy of the system, as well as the tensor function of heat δQ^{ik} :

$$\delta Q^{ik}=V_e\,d(U^{ik}+W^{ik})+\frac{V_e}{c^2}\,d(P_0\,u^i\,u^k)-V_e\,\eta^{ik}\,d\left(\frac{P_0}{1-V^2/c^2}\right), \qquad (15)$$

where V_e is the invariant volume of a small unit of substance or a small volume of space occupied by the field in the absence of substance, U^{ik} and W^{ik} – the stress–energy tensors of gravitational and electromagnetic fields, u^i – 4-velocity of substance, η^{ik} – the metric tensor of Minkowski spacetime.

From (15) it follows that at constant volume V_e of the substance unit the increment of heat occurs from the increments of the density of energy-momentum of fields and changes of the internal pressure P_0, depending on the 4-velocity for an outside observer. All terms listed in (15) can directly increase the kinetic temperature of the substance unit and therefore are part of δQ^{ik}. To obtain the amount of heat of the body as a set of the substance units δQ^{ik} should be summed over all volume elements. Increment of entropy tensor is defined as in (13):

$$dS^{ik} = \frac{\delta Q^{ik}}{T}.$$

Symmetric tensor of entropy is the integral over the volume:

$$S^{ik} = \int \frac{\dfrac{\eta^{ik}\rho_0 c^2}{\sqrt{1-V^2/c^2}} - \rho_0 u^i u^k \sqrt{1-V^2/c^2} - \eta^{ik}\boldsymbol{r}\cdot\left[\nabla(L-P_0)+\rho_0\boldsymbol{G}+\rho_{0q}\boldsymbol{E}\right]}{T} dV_s \,, \qquad (16)$$

where ρ_{0q} is the charge density.

For a unit of substance of gravitationally bound body after a number of simplifications, the formula for the timelike component of the entropy tensor is obtained:

$$S^{00} = -\frac{NR\Delta\rho_0}{\rho_0},$$

where $\Delta\rho_0 \ge 0$ is the change in the density of substance on the length of the unit of substance, R – the gas constant, N – the amount of substance in moles.

We can show that not only S^{00}, but other components of tensor S^{ik} are negative. As it follows from (16), the entropy of the substance unit is proportional to the ratio of the absolute value of ordered energy in this unit and the energy of random thermal motion of particles of substance, taken with the minus sign. Under the ordered energy we mean the energy of directional motion of the substance unit, the energy of pressure compression and the potential energy of the substance unit in gravitational and electromagnetic fields. Entropy is the function of the system state, because if the system state is set by a number of physical quantities, then in each such state, after some relaxation time, usually only one definite relation between the ordered and disordered system energies is carried out that is independent on the way of transition into this state. This relation is fixed by the concept of entropy.

In the theory of infinite hierarchical nesting of matter (Fedosin S G 2009a) it is supposed that the source of ordering and the ordered energy of bodies are the fluxes of gravitons, whose properties are similar to those of photons and neutrinos, as well as high-energy charged particles. These field quanta and particles, appearing at lower levels of matter, due to their relatively high energy in comparison with their mass, have the highest ordering in our world and carry it in the space.

The stream of ordering is received by a gravitational system with a flux of gravitons, and it generates negentropy in the system, as the flux of gravitons outgoing from the system has lower temperature with nearly the same energy as the energy of the incoming flux of gravitons. This negentropy allows reducing the entropy of the system to negative values. In addition, the outgoing emission from the system, typically electromagnetic, has its proper entropy, so that approximately one half of the negentropy of the fluxes of gravitons is spent on the system entropy loss due to outgoing emission.

In accordance with the above-mentioned and (Fedosin S G 2011a), we assume that the observed heating of an object due to gravitational contraction leads to an increase in mass of the object. This process is accompanied by the emission of photons from the object with total energy equal to the relativistic energy of the object, excluding the rest energy, and is equal to the absolute value of the total macroscopic energy (macroscopic binding energy). At the same time the total energy and the entropy of the object have the negative sign. In the reverse process the external radiation heats the object and increases the total macroscopic energy and the entropy, and hence reduces the relativistic rest energy and the mass of the object associated with it.

3.3. Nuclear energy

In modern physics it is supposed that for determination of the relativistic energy of the body it is necessary to sum up the rest energy of its constituent particles and the total energy of the body, taking into account the mechanical energy of particles and the energy of fields. For the fundamental forces the total energy is usually negative, so that the relativistic energy and the body mass are less than the energy and the mass of all particles of the body, separated from each other. In the theory of infinite hierarchical nesting of matter, there is infinite number of levels of the matter with objects of corresponding masses located on them. If at some basic level of matter we take quite many objects and start putting them together into more massive objects, then due to the negative total energy the relative mass of objects will be less and less at each subsequent level of matter, in relation to the total mass of the primary objects.

According to our assumptions, the total energy in the gravitational field is included in the relativistic energy with the negative sign, which leads not to a decrease but to an increase in the relative mass of objects with increasing of the mass of these objects. If we consider the question from a philosophical point of view, the conclusions about the probable decrease or increase in the relative mass of objects as we move to higher levels of matter seem to be equally valid. Apparently, the choice can be made by comparison with the experimental data.

Most clearly the relationship between mass and energy is revealed in the case of fusion of light nuclei and in the decay of massive nuclei, when small differences in the masses of the initial and the final reaction products are accompanied by the release of large amounts of energy. In Table 1, according to (2011 CODATA recommended value; WolframAlpha data), the masses of some nuclei are given in comparison with the sum of the masses of separate protons and neutrons, of which these nuclei could be composed.

Table 1: Masses of some nuclei

Nucleus	Number of neutrons, N_n	Number of protons, N_p	$N_n M_n$, the mass of neutrons, 10^{-27} kg	$N_p M_p$, the mass of protons, 10^{-27} kg	M_N, mass of the nucleus, 10^{-27} kg	$N_n M_n + N_p M_p - M_N$, 10^{-27} kg
$^{2}_{1}H$	1	1	1.674 927 351	1.672 621 777	3.343 583 48	0.003 965 65
$^{62}_{28}Ni$	34	28	56.947 529 93	46.833 409 75	102.808 9	0.972 04
$^{238}_{92}U$	146	92	244.539 393	153.881 203	395.208 8	3.211 8

According to Table 1, the mass of any nucleus is less than the total mass of nucleons, of which the nucleus can be formed. Mass defect, shown in the last column of Table 1 is such that the decrease in the mass of the nucleus can reach almost 1 %. In the standard model it is supposed that after combining the nucleons their total mass decreases due to the negative total energy of the nucleus. If, however, we proceed from our assumptions, then similarly to (3) for the relativistic energy and the mass of the nucleus at rest we should write down:

$$\varSigma_N = -E_{gn} - E_{gp} + E_N , \qquad M_N = \frac{\varSigma_N}{c^2} = -\frac{E_{gn} + E_{gp} - E_N}{c^2} = N_n M_n + N_p M_p + \frac{E_N}{c^2} , \qquad (17)$$

where E_{gn} – total energy of the free neutrons necessary for the formation of the nucleus, E_{gp} – the total energy of the free protons that make up the nucleus, E_N – the total energy of the nucleus in the connection of nucleons, consisting of the kinetic energy of motion and rotation of the nucleons in the nucleus, and of the potential energy of their interaction by means of gravitational and electromagnetic fields in accordance with the gravitational model of strong interaction.

We shall note that in (17) we put the plus sign to the total energy E_N, in contrast to the minus sign, standing before the total energy E_0 in (3). This is due to the fact that after the gravitational contraction the energy of the gravitational field is transferred in the form of radiation to the environment, and to the heating of substance, thus creating the mass of radiation and the additional mass, as it is seen from (3). But the situation with formation of the atomic nucleus from nucleons is different. For the emergence of the nucleus it is necessary either to heat up nucleons from an external source to the temperature sufficient to initiate fusion of the nuclei, or to do some work on the nucleons. While a system emits photons during the gravitational contraction, then in contrast to it for nuclear fusion it is necessary in some way to introduce some extra energy in the system. This is similar of the effect of thermal heating described in the previous section and in our opinion it leads to a decrease in mass of the system.

From a formal point of view, the relation (3) describes the process of creating the mass of photons in the environment of the system and creating the additional mass of the system in the form $\Delta m = -\frac{E_0}{c^2}$. To describe the formation of the nucleus and the changes in its mass we can assume that the interaction between the nucleons leads to the negative mass of photons (photons are not generated, but on the contrary absorbed by the system; or some work is done on the system) and to a certain total energy, taken with the minus sign. Substitution in (3) instead of E_0 the total energy E_N, but taken with the minus sign, gives the change in mass $\Delta m = \frac{E_N}{c^2}$ and the plus sign before E_N in (17). Since the total energy E_N by itself is negative, then in (17) the mass of the nucleus M_N is less than the total mass of protons and neutrons that make up the nucleus.

How are the nucleons held in atomic nuclei? In (Fedosin S G 2009a) we gave some simple models of nuclei and described nuclear forces, due to which the nucleons in a nucleus can be in equilibrium. Similarly, in order to substantiate the stability of some of the hadrons, in (Fedosin S G 2009b) we have developed their models based on the binding of nucleons and light mesons. The solidity of the nuclei is due to the large forces acting between the nucleons of the nucleus. If we assume that the force of attraction due to strong gravitation acts between the nucleons in a nucleus, then there must be also powerful forces of repulsion. These forces arise from the torsion fields of rapidly rotating nucleons. Typically, the force of the torsion field is weaker than the force of gravitational attraction of masses. Similarly, magnetic forces are generally weaker than electrical forces, since in the formula for the magnetic force there is the squared speed of light, which decreases the value of the force. As the magnetic forces, the forces of the torsion field considerably grow at the velocity close to the speed of light, and begin to level off in value with the electric and gravitational forces, respectively. Thus, in order that the spins of the nucleons in a nucleus could effectively repel each other, a very fast rotation of the nucleons is necessary, which generates the field of torsion.

As an illustration, we shall present here a formula for the total energy of deuterium, the simplest nucleus, consisting of a neutron and a proton, according to (Fedosin S G 2009a):

$$E_D = U_g + 2(U - U_p) + \eta U_o + E_r \,, \qquad (18)$$

where $U_g = -\frac{0.26\Gamma M_n M_p}{R}$ is the gravitational energy of the interaction of neutrons and protons (the coefficient 0.26 reflects a decrease in the interaction force due to the high density of substance and is calculated in the upgraded model of gravitation of Fatio-Lesage (Fedosin S G 1999, 2009c) as the consequence of the exponential attenuation of flux of gravitons in substance; at low density of substance this coefficient tends to 1, and the formula for U_g takes Newtonian form),

$\Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514 \cdot 10^{29}$ m^3·kg^{-1}·s^{-2} is the strong gravitational constant according to (Fedosin S G 1999),

e – the elementary electric charge, ε_0 – the vacuum permittivity, M_e – the electron mass, R – the distance between the centers of the neutron and the proton,

$2(U - U_p) = -\frac{83\Gamma(L^2 - L_p^2)}{126\, c_g^2 R_p^3}$ – the change in the energy of the torsion field of strong gravitation of two nucleons, which occurs due to the increase in the spin (the angular momentum) of each nucleon from the value L_p to L,

R_p – the proton radius, approximately equal to the radius of neutron, c_g – the propagation speed of gravitation which is close to the speed of light,

$\eta U_o = \frac{\eta \Gamma L^2}{c_g^2 R^3}$ – the energy of interaction between the spins of two nucleons in their gravitational torsion field,

$\eta = 2.8$ – the coefficient, which reflects an increase in the spin of the nucleons as compared with the value for the angular momentum of the ball in classical physics, and arises as the consequence of taking into account the relativistic rotation, the increase in the mass and the momentum,

$E_r = \frac{L^2 - L_p^2}{I}$ – the increase in the rotational energy of the nucleons during their fusion in the nucleus,

I – the moment of inertia of a nucleon.

Our assumption that the rotation of nucleons in their fusion to the nucleus should be increased, follows from the fact that only in this case, the repulsive force of the spins will be sufficient to counteract the attraction of the nucleons under the influence of strong gravitation. Orientation of nucleon spins in the nucleus of deuterium is of such kind that produces repulsion of the spins, and during the convergence of nucleons due to the equal direction of spins there is an increase in rotation of the nucleons with the increase in the angular momentum because of the effect of gravitational induction. As a result the nucleons start rotating rapidly and reach the maximum possible angular momentum.

For the deuteron the total energy is $E_D = -2.224$ MeV, correspondingly, the binding energy as the absolute value of the total energy is 2.224 MeV. For more massive nuclei with an increased number of protons the formula for the total energy instead of (18) can be written as follows:

$$E_N = U_g + A(U - U_p) + \eta U_o + E_r + W, \qquad (19)$$

where A specifies the number of nucleons in a nucleus,

the gravitational energy U_g, the energy of interaction between the spins ηU_o and the change of the rotational energy E_r are calculated for all the nucleons in the nucleus,

$W = \frac{k z^2 e^2}{4\pi \varepsilon_0 R_N}$ – the electrical energy of protons in the nucleus for the case of their uniform distribution by the volume of the nucleus, when $k \approx 0.6$,

R_N – the average radius of the nucleus, z – the charge number of the nucleus or the number of protons.

In literature, as a rule specific binding energy, or the absolute value of the total energy per nucleon are considered, i.e., the quantity $\frac{|E_N|}{A}$, and its dependence on A is built. For light nuclei the main contribution to (19) is made by the energy of strong gravitation U_g. Assuming that radius of the nucleus is approximated by the usual formula $R_N = R_0 A^{1/3}$, where $R_0 = (1.2 - 1.5)\cdot 10^{-15}$ m, and the mass of the nucleus $M_N \approx A M_p$, we can write as in (Fedosin S G 2009a) a proportional relation:

$$\frac{|E_N|}{A} \sim \frac{|U_g|}{A} = \frac{k \Gamma M_N^2}{A R_N} \sim A^{2/3}.$$

This dependence describes well the growth of the specific binding energy of nuclei up to $A \approx 20$. Then saturation of the energy of strong gravitational energy takes place, the energy of the nucleus changes not proportionally to the square of nuclear mass, but much weaker. As it was shown in (Fedosin S G 2009c), the cross section of interaction of gravitons with nucleons is such that it is enough to put three nucleons in the way of the flux of gravitons in order to significantly reduce the flux (approximately 2.718 times, this number is the base of natural logarithms). When the number of nucleons in the nucleus is more than 17–23 then addition of new nucleons increases less and less the gravitational energy per nucleon.

At the same time adding protons to the nucleus with increasing of the mass and the charge of the nucleus leads to a marked increase in the positive electric energy which begins to compensate the change of the negative gravitational energy. As a result, at $A = 62$ for ${}^{62}_{28}\mathrm{Ni}$ the maximum of the dependence $\frac{|E_N|}{A}$ on A is achieved, and then the specific binding energy begins to decrease with the increase in A. Thus, the formulas for the strong gravitation and for the electromagnetic forces and energies can describe the equilibrium of nucleons in the nucleus, and also explain the dependence of the specific binding energy on the mass number. The decrease in the mass of the atomic nuclei, compared with a sum of the masses of the constituent nucleons, is the consequence of the opposite fluxes of energy necessary for the emergence of the binding energy, compared with the case of the ordinary gravitational contraction of matter.

3.4. General theory of relativity

The axiomatics of general theory of relativity (GTR) is associated with recognition of the gravitational field as some form of metric field, and with geometric difference between a curved Riemannian spacetime and the flat Minkowski spacetime. Currently, GTR is the most famous and developed theory of gravitation. As the basis of the theory the Hilbert-Einstein equations for the metric can be considered:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi\gamma}{c^4} \left(\phi_{\mu\nu} + W_{\mu\nu} \right), \qquad (20)$$

where $R_{\mu\nu}$ − Ricci tensor, R − the scalar curvature, $g_{\mu\nu}$ − the metric tensor, Λ − the cosmological constant, γ − the gravitational constant, c − the speed of light, $\phi_{\mu\nu}$ − the stress–energy tensor of substance, $W_{\mu\nu}$ − the stress–energy tensor of electromagnetic field and other non-gravitational fields.

If we ignore the cosmological constant and consider the metric around a spherical, uncharged, non-rotating mass with the density of its substance ρ_0 and the tensor $\phi_{\mu\nu} = \rho_0 u_\mu u_\nu$, where u_μ is 4-velocity, then in spherical 4-coordinates $x^0 = ct, \quad x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi$, the metric tensor as the solution of equation (20) has the following components:

$$g_{\mu\nu} = \left| \begin{array}{cccc} 1 - \frac{2\gamma M}{rc^2} & 0 & 0 & 0 \\ 0 & -\frac{1}{1 - \frac{2\gamma M}{rc^2}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{array} \right| . \qquad (21)$$

This is the well-known Schwarzschild solution for the metric around a massive point body with mass M, which depends only on the angle θ and the distance r between the attracting center and observation point.

The equation of motion of GTR for the test body around the attractive mass M is as follows:

$$\frac{d}{ds}\left(\frac{dx^\nu}{ds} \right) + \Gamma^\nu_{\mu\rho} \frac{dx^\mu}{ds} \frac{dx^\rho}{ds} = 0, \qquad (22)$$

where $ds = c\, d\tau$ is the invariant interval, $d\tau$ − the differential of the proper time of the test body, dx^ν − 4-vector of the test body displacement, $\Gamma^\nu_{\mu\rho} = \frac{1}{2} g^{\nu\sigma} (\partial_\mu g_{\sigma\rho} + \partial_\rho g_{\sigma\mu} - \partial_\sigma g_{\mu\rho})$ − Christoffel symbol, which is expressed through the metric tensor and its derivatives with respect to the coordinates.

If we use the metric tensor (21) to solve equation (22) for the timelike component dx^ν, when $\nu = 0$, we obtain the following:

$$\frac{d}{d\tau}\left(c\, g_{00} \frac{dt}{d\tau} \right) = 0 . \qquad (23)$$

We shall multiply (23) by the value mc, where m is mass of the test body, and look at the situation at infinity. Here g_{00} tends to 1 because of the large value r, and the differential of the proper time has the same form as in special theory of relativity: $d\tau = dt\sqrt{1-V_\infty^2/c^2}$, where V_∞ denotes the speed of the test body at infinity. Then (23) becomes equality for infinity:

$$\frac{1}{\sqrt{1-V_\infty^2/c^2}}\frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-V_\infty^2/c^2}}\right)=0.$$

In the brackets of the equality we have the relativistic energy of the body with the mass m, which is moving at infinity at the speed V_∞. Consequently, (23) can be treated as the law of conservation of energy of the test body in the gravitational field (in free fall the energy of gravitational field is converted into the kinetic energy, and the sum of the negative energy of the field and the positive kinetic energy is zero). After multiplying (23) by the value mc and integrating we obtain the relativistic energy:

$$\Sigma_m = mc^2 g_{00}\frac{dt}{d\tau} = \frac{mc^2}{\sqrt{1-V_\infty^2/c^2}} = const. \qquad (24)$$

According to (24), during the fall of the test body to the attractive center and changing of the radial distance r the value $g_{00} = 1-\frac{2\gamma M}{rc^2}$ changes, as well as the differential of the proper time $d\tau$ with respect to the differential of the coordinate time dt, but the relativistic energy of the test body remains unchanged.

We shall assume now that the particles of the substance of the test body at infinity were once scattered in such a way that their speed V_∞ was near zero, and then the particles will approach a massive body and collide with each other. If in the collision the particles lose part of their total angular momentum, and convert part of their energy into the thermal energy of the collision E_f, which is emitted from the system, then a stationary rotation of substance around the center of attraction is possible. The condition for this is the satisfying of the virial theorem, according to which the absolute value of the total energy of the system must be equal to the energy emitted from systems: $|E_m| = E_f$. As a result the relativistic energy of the test body, falling from infinity at zero initial velocity to the source of gravitational field, will decrease by the amount E_f:

$$\Sigma = \Sigma_m - E_f = \Sigma_m - |E_m| = mc^2 + E_m. \qquad (25)$$

Thus, in general theory of relativity the substance of the mass m, rotating in a stationary state around the center of attraction, must reduce its relativistic energy due to the contribution of the negative total energy E_m.The same conclusion will be valid, if the attractive center arises due to the collapse of a massive cloud of substance, which reduces in the course of time its angular momentum by means of electromagnetic radiation. Equation (25) by its meaning does not coincide with (3), in which the total energy E_0 is not added but subtracted from the rest energy.

3.5. Covariant theory of gravitation

In contrast to the general theory of relativity, the covariant theory of gravitation (CTG) is based on the axioms of Lorentz-invariant theory of gravitation (Fedosin S G 2009a, 2011b), and is a covariant generalization to the curved Riemannian spacetime. Gravitation in CTG is considered not fictitious geometric, but an actual physical force, and can be substantiated using Fatio-Le Sage's theory of gravitation. In CTG the substance through a 4-vector of density of momentum J^μ generates a gravitational field with a 4-potential D^μ, satisfying the wave equation in the Riemannian spacetime:

$$\Box^2 D^\mu = \frac{\partial^2 D^\mu}{c_g^2\,\partial t^2} - \nabla^2 D^\mu + R^\mu{}_\nu D^\nu = -\frac{4\pi\gamma J^\mu}{c_g^2}, \qquad (26)$$

where c_g – the propagation speed of gravitation which is close to the speed of light,

$\Box^2$ is 4-d'Alembert operator, $R^\mu{}_\nu$ is the Ricci tensor with mixed indices, γ − the gravitational constant.

4-vector of density of momentum J^μ is determined by the product of the density of substance ρ_0, found in the frame of reference of the substance unit at rest, and the 4-velocity: $J^\mu = \rho_0 u^\mu$. If we use the approximation of weak field and small velocities, when the CTG is transformed into Lorentz-invariant theory of gravitation, the 4-velocity is as follows:

$$u^\mu = \frac{dx^\mu}{d\tau} \approx \left(\frac{c_g}{\sqrt{1-V^2/c_g^2}}, \frac{\boldsymbol{V}}{\sqrt{1-V^2/c_g^2}} \right). \qquad (27)$$

The same expression of the 4-velocity (27) with condition $c_g = c$ is adopted in general relativity for the case of the weak field and small velocities. In Riemannian space we can introduce (Fedosin S G 2009a) the operator of differentiation with respect to the proper time τ :

$$\frac{D}{D\tau} = u^\nu \nabla_\nu , \qquad (28)$$

where the symbol D denotes the total differential in curved spacetime, ∇_ν is the covariant derivative.

When the operation of the covariant antisymmetric tensor product of the covariant gradient operator and the covariant 4-vector potential D_μ is used the gravitational field strength tensor has the form:

$$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu ,$$

In view of $\Phi_{\mu\nu}$ the relationship between the substance and the field (26) is as follows:

$$\frac{4\pi\gamma J^\mu}{c_g^2} = \nabla_\nu \Phi^{\mu\nu}. \qquad (29)$$

The covariant 4-vector of potential is defined as:

$$D_\mu = \left(\frac{\psi}{c_g}, -\boldsymbol{D} \right),$$

where ψ – the scalar potential, $\boldsymbol{D}$ – the vector potential.

The intrinsic properties of gravitational field strengths, independent on the material sources, are set by the relation:

$$\nabla_\rho \Phi_{\mu\nu} + \nabla_\mu \Phi_{\nu\rho} + \nabla_\nu \Phi_{\rho\mu} = \partial_\rho \Phi_{\mu\nu} + \partial_\mu \Phi_{\nu\rho} + \partial_\nu \Phi_{\rho\mu} = 0. \qquad (30)$$

Relations (29) and (30) have the form in which equations of gravitational field of CTG are covariant in any frame of reference.

The field in its turn affects the substance, creating a gravitational force. 4-vector of density of the gravitational force is defined by:

$$f_g^{\,\mu} = \Phi^\mu{}_\nu J^\nu = -\nabla_\nu U^{\mu\nu}, \qquad (31)$$

where $U^{\mu\nu} = \frac{c_g^2}{4\pi\gamma}\left(-g^{\mu\rho}\,\Phi_{\rho\sigma}\,\Phi^{\sigma\nu} + \frac{1}{4} g^{\mu\nu}\,\Phi^{\sigma\rho}\,\Phi_{\rho\sigma} \right)$ is the stress–energy tensor constructed with the help of the tensor of strengths of the gravitational field $\Phi_{\mu\nu}$ and equations (29) – (31). The presence of the tensor $U^{\mu\nu}$ distinguishes CTG from the general relativity, in which an exact expression for the stress–energy tensor of gravitational field is absent.

The general definition of force in CTG is found by means of (28):

$$f^{\mu}=\frac{DJ^{\mu}}{D\tau}=u^{\nu}\nabla_{\nu}J^{\mu}=u^{\nu}(\partial_{\nu}J^{\mu}+\Gamma^{\mu}_{\nu\rho}J^{\rho})=\frac{dJ^{\mu}}{d\tau}+\Gamma^{\mu}_{\nu\rho}u^{\nu}J^{\rho}. \qquad (32)$$

The electromagnetic force is defined by:

$$\underset{e}{f^{\mu}}=F^{\mu}_{\ \nu}\,j^{\nu}=-\nabla_{\nu}W^{\mu\nu},$$

where $F^{\mu}_{\ \nu}$ – the electromagnetic tensor, $j^{\nu}=\rho_{0q}u^{\nu}$ – 4-current, ρ_{0q} – the charge density in the reference frame where the charge is at rest, $W^{\mu\nu}$ – the electromagnetic stress–energy tensor.

If there are only two fundamental fields, gravitational and electromagnetic, which create forces, then the equation of motion of the substance unit takes the form:

$$\frac{dJ^{\mu}}{d\tau}+\Gamma^{\mu}_{\nu\rho}u^{\nu}J^{\rho}=-\nabla_{\nu}U^{\mu\nu}-\nabla_{\nu}W^{\mu\nu}. \qquad (33)$$

As it was shown in (Fedosin S G 2011b), the equation of motion in general relativity is derived from (33) as a special case.

To determine the spacetime metric the Hilbert-Einstein equations (Fedosin S G 2012a) are used:

$$R_{\mu\nu}-\frac{R}{2}g_{\mu\nu}=\frac{8\pi\gamma\beta}{c_g^4}\left(\phi_{\mu\nu}+U_{\mu\nu}+W_{\mu\nu}\right). \qquad (34)$$

In contrast to (20), in CTG the gravitational field, along with the electromagnetic field is involved in obtaining the metric, so the right side of (34) contains the stress–energy tensor $U_{\mu\nu}$ of gravitational field. The stress–energy tensor of substance $\phi_{\mu\nu}$ in CTG is constructed so that the covariant derivative of this tensor, taken with contravariant indices, would give the force density (32): $f^{\mu}=\nabla_{\nu}\phi^{\mu\nu}$. If we take the covariant derivative of (34), the left side vanishes because of the properties of the metric tensor. This again gives the equality for the density of forces (33):

$$f^{\mu}=\nabla_{\nu}\phi^{\mu\nu}=-\nabla_{\nu}U^{\mu\nu}-\nabla_{\nu}W^{\mu\nu}=\frac{dJ^{\mu}}{d\tau}+\Gamma^{\mu}_{\nu\rho}u^{\nu}J^{\rho}.$$

The solution of the equation for the metric (34) around an uncharged ball at rest gives the components of the metric tensor in spherical 4-coordinates $x^0=ct,\quad x^1=r,\quad x^2=\theta,\quad x^3=\varphi$, (Fedosin S G 2009a):

$$g_{\mu\nu}=\begin{vmatrix} 1+\frac{\gamma M\alpha}{rc^2}-\frac{\beta\gamma^2M^2}{r^2c^4} & 0 & 0 & 0 \\ 0 & -\frac{1}{1+\frac{\gamma M\alpha}{rc^2}-\frac{\beta\gamma^2M^2}{r^2c^4}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{vmatrix}. \qquad (35)$$

The coefficients α and β in (35) from equations (34) are not defined and should be specified for each particular system of bodies.

Using the metric tensor (35), we can find the solution to the equations of motion (33) for the timelike component J^{μ}, when $\mu=0$. In the case of the weak gravitational field and at constant density ρ_0 of the substance unit we obtain:

$$\frac{d}{d\tau}\left(g_{00}\frac{dt}{d\tau}\right)=-\frac{\gamma M}{c^2 r^2}\frac{dr}{d\tau}, \qquad \text{or} \qquad g_{00}\frac{dt}{d\tau}=\frac{\gamma M}{c^2 r}+C_1. \qquad (36)$$

At infinity g_{00} tends to 1, $d\tau = dt\sqrt{1-V_\infty^2/c^2}$, where V_∞ denotes the velocity of the test body at infinity, and $C_1=\frac{1}{\sqrt{1-V_\infty^2/c^2}}$. We shall suppose $V_\infty = 0$, then, after multiplication by mc^2 (36) can be written as follows:

$$\Sigma_m = mc^2 g_{00}\frac{dt}{d\tau}=mc^2+\frac{\gamma M m}{r}=mc^2-U. \qquad (37)$$

According to (37) the substance, which had at infinity the relativistic energy mc^2, during a fall in the gravitational field increases its energy by the value equal to the absolute value of the potential energy of the field $U=-\frac{\gamma M m}{r}$. Although in CTG there is difference of expressions g_{00} and $d\tau$ from the corresponding expressions in general relativity, in (37) an approximate equality between the absolute value of change of the potential energy of gravitational field and the change in the kinetic energy of substance motion is satisfied.

If for this system the virial theorem is valid, for which the decrease in the angular momentum of the falling substance, emission of energy E_f from the system and increase in the kinetic energy of the substance by the value $E_k = E_f$ are required, then the relativistic energy is equal to:

$$\Sigma = \Sigma_m - E_f = mc^2 - U - E_k = mc^2 - E_m, \qquad (38)$$

where $E_m = U + E_k$ is the total energy of mass m in gravitational field.

If the gravitation is created by a stationary system with a mass m, then the energy E_m in (38) will characterize the change in the relativistic energy of the system that has occurred due to the action of gravitational field, the interaction of particles of substance and emission from the system. Relation (38) has the same form as (3), where before the total energy E_0 there is a negative sign. We can see that difference between the results of CTG and general relativity is due to difference in the equations of motion (33) and (22).

4. CONCLUSION

Having examined some cases of mass-energy relation, we made the assumption that if the system loses energy in the form of emission or the work is done on the surrounding bodies, then the total energy of the particles of the system must be subtracted from the rest energy of the particles constituting the system. For fundamental forces the total energy is negative, which leads to an increase in the mass of the particles system as compared with the sum of the masses of the particles separately. In particular, the mass of a star in accordance with the covariant theory of gravitation can be larger than the total mass of fragments of stellar substance. This is confirmed in (Fedosin S G 2012b). In another case, when for the formation of the system it is necessary to add energy to it or to do work on it, the total energy of the particles in the system should be added to the rest energy of the particles constituting the system. In some cases this leads to the decrease in the relativistic energy and the mass of the system (an example is the formation of nuclei of the nucleons).

Our assumptions are essentially the opposite to the standard view, for which a suitable form of the total energy is always just added to the rest energy of the particles constituting the system. In the general theory of relativity as for the stars and so for the atomic nuclei, this leads to a decrease in their mass as compared with the rest mass of the particles constituting these objects, and heating of the body increases its mass. Apparently, in such situation additional confirmation is required, whether in fact there is increase, or decrease in the inert and gravitational masses of massive complex objects as compared with the sum of the masses of their parts.

In this connection, we should consider the following. If we calculate the share of the gravitational binding energy in relation to the rest energy of the substance for a typical neutron star, this share could reach 6 %. The same value is expected for increase (or decrease) in the gravitational mass of the star, and hence in the force acting on the test body near the star. On the other hand, the force acting on the body, according to the Fatio-Le Sage's theory of gravitation, depends also on the density of the body. For two bodies of low density the law of Newton's gravitational force is satisfied with sufficient accuracy, but when the substance density of the interacting bodies reaches the density of neutron stars, the force decreases in magnitude and is equal to 26 % of the Newtonian force (Fedosin S G 2009a). As it can be seen, the effect of changing of the gravitational mass can depend not only on the total energy of bodies, but also on other parameters, which can make the experimental verification of the theory more complicated.

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